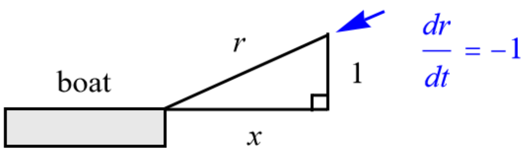
Exercise 22

A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s, how fast is the boat approaching the dock when it is 8 m from the dock?

Solution

Draw a schematic of the triangle formed by the rope at some time.



Note that dr/dt is negative because the hypotenuse decreases as time goes on. r and x are related by the Pythagorean theorem.

$$r^2 = x^2 + 1^2$$

Take the derivative of both sides with respect to time by using the chain rule.

$$\frac{d}{dt}(r^2) = \frac{d}{dt}(x^2 + 1^2)$$

$$2r \cdot \frac{dr}{dt} = 2x \cdot \frac{dx}{dt}$$

$$r\frac{dr}{dt} = x\frac{dx}{dt}$$

Solve for dx/dt, the rate that the distance to the dock is increasing with respect to time.

$$\frac{dx}{dt} = \frac{r}{x}\frac{dr}{dt}$$

$$= \frac{\sqrt{x^2 + 1^2}}{x}\frac{dr}{dt}$$

$$= \frac{\sqrt{x^2 + 1}}{x}(-1)$$

Therefore, when the boat is 8 m from the dock,

$$\left.\frac{dx}{dt}\right|_{x=8} = \frac{\sqrt{(8)^2 + 1}}{(8)}(-1) = -\frac{\sqrt{65}}{8} \approx -1.00778\ \frac{\text{m}}{\text{s}}.$$

The boat's approaching the dock at about a meter per second when it's 8 meters away.